Math 140, Course Review

Questions 1-48 are review for the Midterm
All 83 Questions are review for the Final Exam

Find the domain and range of the function.
1) \( f(x) = -6 + \sqrt{x} \)
   Objective: (1.1) Find the Domain and Range of a Function

Find the domain and graph the function.
2) \( F(t) = \frac{t + 4}{t + 4} \)
   Objective: (1.1) Find Domain and Graph Function

Solve the problem.
3) If \( f(x) = \frac{1}{x} \) and \( g(x) = 7x^2 \), find \( g(f(x)) \).
   Objective: (1.1) Compose Functions

Determine if the function is even, odd, or neither.
4) \( f(x) = -8x^5 - 3x^3 \)
   Objective: (1.1) Determine If Function Is Even, Odd, or Neither
Graph the function.

5) \( G(x) = \begin{cases} x + 3, & x < 0 \\ 3, & x \geq 0 \end{cases} \)

Objective: (1.2) Graph Piecewise Function

Find a formula for the function graphed.

6) ____________

Objective: (1.2) Write Formula for Piecewise Function
Solve the problem.

7) The accompanying figure shows the graph of $y = -x^2$ shifted to a new position. Write the equation for the new graph.

![Graph of $y = -x^2$ shifted to a new position.]

Objective: (1.2) Use Transformations to Find Graph

State the period of the function and graph.

8) $\sin \left( x + \frac{\pi}{4} \right) - 2$

![Graph of $\sin \left( x + \frac{\pi}{4} \right) - 2$.]

Objective: (1.4) Graph Trigonometric Function (Sine, Cosine)
Graph the function.

9) \( f(x) = \frac{x}{x+2} \)

Objective: (1.8) Graph Rational Function

Find the average velocity (slope of the secant line) of the function over the given interval.

10) \( y = x^2 + 7x, [6, 9] \)

Objective: (2.1) Calculate Average Velocity

Find the instantaneous velocity (slope of the tangent line) for the function at the given value of \( x \).

11) \( y = x^2 + 5x, x = 4 \)

Objective: (2.1) Calculate Slope of Tangent Line

Use the graph to evaluate the limit.

12) \( \lim_{{x \to 0}} f(x) \)

Objective: (2.2) Find Limits from a Graph

Find the limit.

13) \( \lim_{{x \to -10}} (2x - 2) \)

Objective: (2.3) Evaluate Limits of Linear Functions
Give an appropriate answer.

14) Let \( \lim_{x \to 5} f(x) = 8 \) and \( \lim_{x \to 5} g(x) = 7 \). Find \( \lim_{x \to 5} [f(x) \cdot g(x)] \).

Objective: (2.3) Apply Limit Laws.

Find the limit, if it exists.

15) \( \lim_{x \to -5} \frac{x^2 + 7x + 10}{x + 5} \)

Objective: (2.3) Find Limits of Functions (Denom \( \neq 0 \))

Find all vertical asymptotes of the given function.

16) \( g(x) = \frac{x + 9}{x^2 - 4} \)

Objective: (2.4) Find Vertical Asymptotes of Function

Find the limit.

17) \( \lim_{x \to -2} \frac{1}{x + 2} \)

Objective: (2.4) Evaluate Limits Analytically

18) \( \lim_{x \to -\infty} \frac{-18x^2 + 9x + 7}{-8x^2 - 4x + 19} \)

Objective: (2.5) Evaluate Limit at Infinity I

19) \( \lim_{x \to -\infty} \frac{\cos 4x}{x} \)

Objective: (2.5) Evaluate Limit at Infinity I

Find all horizontal asymptotes of the given function, if any.

20) \( h(x) = \frac{4x^3 - 3x}{8x^3 - 9x + 9} \)

Objective: (2.5) Find Horizontal Asymptotes of Function

Find the limit, if it exists.

21) \( \lim_{x \to 4} \sqrt{x - 9} \)

Objective: (2.6) Find Limit of Radical Expression

Determine the value(s) for which the function is not continuous.

22) \( y = \frac{5}{x^2 - 4} \)

Objective: (2.6) Find Intervals of Continuity
Evaluate \[ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \] for the given x and function f.

23) \( f(x) = 4x^2 - 3 \) for \( x = 4 \)  

Objective: (1.4) Find Limit of Average Rate of Change of Function

Find the slope of the curve at the given point.

24) \( y = x^2 + 9x, \ x = 3 \)  

Objective: (3.1) Find Slope of Tangent Line at Given Point

Calculate the derivative of the function. Then find the value of the derivative as specified.

25) \( g(x) = x^3 + 5x; \ g'(1) \)  

Objective: (3.1) Find Derivative and Evaluate at Point

Find an equation for the tangent to the curve at the given point.

26) \( y = x^2 - x, \ (-4, 20) \)  

Objective: (3.1) Find Equation of Tangent to Curve at Point

Find the second derivative.

27) \( s = \frac{13t^3}{3} + 13 \)  

Objective: (3.2) Find Second Derivative

Find \( y' \).

28) \( y = (4x - 4)(3x^3 - x^2 + 1) \)  

Objective: (3.2) Find Derivative of Product

Find the derivative of the function.

29) \( y = \frac{x^3}{x - 1} \)  

Objective: (3.3) Find Derivative of Quotient

Find the derivative.

30) \( s = \frac{7e^t}{2e^t + 1} \)  

Objective: (3.3) Find Derivative of Exponential

31) \( y = \frac{5}{x} + 9 \sec x \)  

Objective: (3.4) Find Derivative of Trigonometric Function

Find the indicated derivative.

32) Find \( y'' \) if \( y = 5 \sin x \).  

Objective: (3.4) Find Second Derivative of Trigonometric Function
The function \( s = f(t) \) gives the position of a body moving on a coordinate line, with \( s \) in meters and \( t \) in seconds.

33) \( s = 2t^2 + 4t + 4, \ 0 \leq t \leq 2 \)

Find the body’s displacement and average velocity for the given time interval.

Objective: (3.5) Find Displacement/Velocity/Speed/Accel Given Position Function

34) \( s = 6t^2 + 3t + 7, \ 0 \leq t \leq 2 \)

Find the body’s speed and acceleration at the end of the time interval.

Objective: (3.5) Find Displacement/Velocity/Speed/Accel Given Position Function

The equation gives the position \( s = f(t) \) of a body moving on a coordinate line (\( s \) in meters, \( t \) in seconds).

35) \[
\begin{array}{c|c|c}
\text{t (sec)} & 1 & 2 \\
\hline
\text{s (m)} & 1 & 0 \\
\end{array}
\]

When is the body standing still?

Objective: (3.5) Analyze Motion Depicted in Graph

Solve the problem.

36) At time \( t \), the position of a body moving along the \( s \)-axis is \( s = t^3 - 21t^2 + 120t \) m. Find the body’s acceleration each time the velocity is zero.

Objective: (3.5) Solve Apps: Motion

Find the derivative of the function.

37) \( q = \sqrt{10r - r^7} \)

Objective: (3.6) Find Derivative Using Chain Rule I

38) \( y = (1 + 5x)e^{-5x} \)

Objective: (3.6) Find Derivative Using Chain Rule I

Find \( dy/dt \).

39) \( y = \cos^4(\pi t - 17) \)

Objective: (3.6) Find Derivative Using Chain Rule II

Use implicit differentiation to find \( dy/dx \).

40) \( x^3 + 3x^2y + y^3 = 8 \)

Objective: (3.7) Use Implicit Differentiation to Find Derivative
At the given point, find the slope of the curve or the line that is tangent to the curve, as requested.

41) \( y^4 + x^3 = y^2 + 9x \), slope at (0, 1)  
   **Objective:** (3.7) Use Implicit Differentiation to Find Slope or Tangent Line

Find the value of \( df^{-1}/dx \) at \( x = f(a) \).

42) \( f(x) = 2x + 9 \), \( a = 5 \)  
   **Objective:** (3.9) Find the Value of Derivative of Inverse at Given Point

43) \( f(x) = 5x^2 \), \( x \geq 0 \), \( a = 5 \)  
   **Objective:** (3.9) Find the Value of Derivative of Inverse at Given Point

Find the derivative of \( y \) with respect to \( x \), \( t \), or \( \theta \), as appropriate.

44) \( y = \ln 8x^2 \)  
   **Objective:** (3.8) Find Derivative of Natural Logarithm Function

Find the indicated tangent line.

45) Find the tangent line to the graph of \( f(x) = e^{3x} \) at the point (0, 1).  
   **Objective:** (3.8) Find Tangent Line of General Exponential Function

Find the derivative of \( y \) with respect to \( x \).

46) \( y = \cos^{-1} (11x^2 - 4) \)  
   **Objective:** (3.9) Find Derivative of Inverse Trig Function

47) \( y = \sin^{-1} (e^{4t}) \)  
   **Objective:** (3.9) Find Derivative of Inverse Trig Function

Solve the problem.

48) Water is falling on a surface, wetting a circular area that is expanding at a rate of 8 \( \text{mm}^2/s \). How fast is the radius of the wetted area expanding when the radius is 179 mm? (Round your answer to four decimal places.)  
   **Objective:** (3.10) Solve Apps: Related Rates: Distance
Determine from the graph whether the function has any absolute extreme values on the interval \([a, b]\).

\[ f(x) = x^3 - 9x^2 + 3 \]

Find the absolute extreme values of the function on the interval.

\[
\begin{align*}
\text{Increasing} & \quad f(x) = \frac{1}{4}x^2 - \frac{1}{2}x \\
\end{align*}
\]

Using the derivative of \( f(x) \) given below, determine the intervals on which \( f(x) \) is increasing or decreasing.

\[ f'(x) = (3 - x)(5 - x) \]

Find the largest open interval where the function is changing as requested.

\[ y'': \begin{array}{c|c|c|c|c}
\text{Interval} & -2 & 0 & 2 & \text{Sign of } y'' \\
\hline
\text{Sign} & + & + & + & - \\
\end{array} \]

Solve the problem.

\[ y''' : \begin{array}{c|c|c|c|c}
\text{Interval} & -1 & 1 & \text{Sign of } y''' \\
\hline
\text{Sign} & + & - & - \\
\end{array} \]

Objective: (4.1) Determine if a Graph Exhibits Absolute Extrema
Objective: (4.1) Find Critical Points
Objective: (4.1) Find Values and Locations of Absolute Extrema I
Objective: (3.3) Analyze \( f(x) \) Given \( f'(x) \): Determine Monotonic Intervals
Objective: (4.2) Determine Intervals of Increase and Decrease
Objective: (4.2) Sketch Functions Given Properties
Use the graph of the function $f(x)$ to locate the local extrema and identify the intervals where the function is concave up and concave down.

55) 

Solve the problem.

56) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

Objective: (4.4) Solve Apps: Geometry

57) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 43.5 ft$^3$. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

Objective: (4.4) Solve Apps: Geometry

58) The velocity of a particle (in ft/s) is given by $v = t^2 - 3t + 2$, where $t$ is the time (in seconds) for which it has traveled. Find the time at which the velocity is at a minimum.

Objective: (3.1) Solve Apps: Extreme Values

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

59) $f(x) = x + \frac{27}{x}$, $[3, 9]$ 

Objective: (4.6) Find Points Guaranteed to Exist by the Mean Value Theorem

Use l'Hopital's Rule to evaluate the limit.

60) $\lim_{x\to1} \frac{x^3 - 4x^2 + 3}{x - 1}$ 

Objective: (4.7) Evaluate Limits of the Form 0/0
\[ \lim_{x \to \infty} x \sin \frac{2}{x} \]

Objective: (4.7) Evaluate Limits of the Form \( \frac{\infty}{\infty}, 0 \cdot \infty \), or \( \infty \cdot \infty \)

Find an antiderivative of the given function.

62) \( 9x^2 + 16x - 4 \)

Objective: (4.8) Find General Antiderivatives

63) \( 8 \cos 9x \)

Objective: (4.8) Find General Antiderivatives

Solve the initial value problem.

64) \( \frac{dy}{dx} = \frac{1}{x^3} + x, \ x > 0; \ y(2) = 5 \)

Objective: (4.8) Solve Initial Value Problems

65) \( \frac{ds}{dt} = \cos t - \sin t, \ s\left(\frac{\pi}{2}\right) = 9 \)

Objective: (4.8) Solve Initial Value Problems

Find the most general antiderivative.

66) \( \int \left[ 5t^2 + \frac{t}{7} \right] \, dt \)

Objective: (4.8) Find General Indefinite Integrals I

Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

67) \( f(x) = \frac{1}{x} \) between \( x = 2 \) and \( x = 8 \) using a right sum with two rectangles of equal width.

Objective: (5.1) Approximate Area Using Finite Sum

Express the sum in sigma notation.

68) \( 1 - 4 + 16 - 64 + 256 \)

Objective: (5.1) Express Sum in Sigma Notation

Evaluate the sum.

69) \( \sum_{k=1}^{6} k^2 - 8 \)

Objective: (5.1) Evaluate Sum Given Sigma Notation
Graph the function \( f(x) \) over the given interval. Partition the interval into 4 subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum \( \sum_{k=1}^{4} f(c_k) \Delta x_k \), using the indicated point in the kth subinterval for \( c_k \).

70) \( f(x) = 2x + 4 \), \([0, 2]\), left-hand endpoint

Graph the integrand and use geometry to evaluate the integral.

71) \( \int_{4}^{8} x \, dx \)

Solve the problem.

72) Suppose that \( \int_{7}^{8} f(x) \, dx = -2 \). Find \( \int_{7}^{8} 5f(u) \, du \) and \( \int_{7}^{8} -f(u) \, du \).

Find the derivative.

73) \( y = \int_{0}^{x^{10}} \cos \sqrt{t} \, dt \)

Evaluate the integral.

74) \( \int_{-2}^{6} 6x^5 \, dx \)

75) \( \int_{0}^{16} 2\sqrt{x} \, dx \)

Objective: (5.1) Construct Rectangles for Riemann Sum

Objective: (5.2) Use Net Area/Geometry to Evaluate Integral

Objective: (5.2) Use Properties/Known Values to Find Integral

Objective: (5.3) Differentiate Integral

Objective: (5.3) Evaluate the Definite Integral
76) \[
\int_{0}^{\pi} \frac{9 - \sin 4x}{4} \, dx
\]

Objective: (5.3) Evaluate the Definite Integral

Find the area of the shaded region.

Objective: (5.3) Find Area of Shaded Region on Graph

Find the total area of the region between the curve and the x-axis.

\( y = x^2 - 6x + 9; \ 2 \leq x \leq 4 \)

Objective: (5.3) Find Area Between Curve and x-Axis

Use symmetry to evaluate the integral.

\[
\int_{-\pi/2}^{\pi/2} \cos^5 8x \, dx
\]

Objective: (5.4) Use Symmetry to Evaluate Integral

Find the average value of the function over the given interval.

\( f(x) = 4 - x \) on \([0, 4]\)

Objective: (5.4) Find Average Value of Function

\( y = x^2 - 2x + 6; \ [0, 2] \)

Objective: (5.4) Find Average Value of Function

Evaluate the integral.

\[
\int \frac{x \, dx}{(7x^2 + 3)^5}
\]

Objective: (5.5) Use Substitution Rule to Evaluate Indefinite Integral I

Use the substitution formula to evaluate the integral.

\[
\int_{0}^{1} \frac{4 \, r \, dr}{\sqrt{4 + 2r^2}}
\]

Objective: (5.5) Use Substitution Formula to Evaluate Definite Integral I